AD-A102 700   ARMY ENGINEER TOPOGRAPHIC LABS FORT BELVOIR VA   F/G 17/5
ON THE INTERPOLATION OF GRAVITY ANOMALIES AND DEFLECTIONS OF TH--ETC(U)
SEP 81 H B VON LUETZOW
UNCLASSIFIED ETL-R015   NL

1 OF 1
AD
A102700

END
DATE
FILMED
9-81
DTIC

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ETL-R015	2. GOVT ACCESSION NO. AD-A102700	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) On the Interpolation of Gravity Anomalies and Deflections of the Vertical in Mountainous Terrain		5. TYPE OF REPORT & PERIOD COVERED Paper
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. Baussus von Luetzow		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Engineer Topographic Laboratories Ft. Belvoir, VA 22060		12. REPORT DATE 5 Sep 1981
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) LEVEL II		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

DTIC ELECTE AUG 11 1981 S D C

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

interpolation
gravity anomalies
deflections of the vertical
mountainous terrain

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The paper first addresses the interpolation of gravity anomalies in mountainous terrian,to be represented as the sum of a "signal" variable with a quasi-stationary, estimation structure and a computable "noise" variable without a stationary character. It then develops the particular solution of the boundary value problem of physical geodesy which permits a similar representation of deflections of the vertical and draws some conclusions concerning the inapplicability of Molodensky's series approach and of the collocation methodfor the accurate determination of vertical deflections from unmodified gravity anomalies in

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

403112

Block #20 cont.

mountainous terrain. Therafter, it discusses the estimation of signal-type deflections of the vertical by means of spatial covariance functions, i.e., by a linear regression technique called statistical collocation in physical geodesy, and provides first order expansions of planar covariance functions.

ON THE INTERPOLATION OF GRAVITY ANOMALIES AND DEFLECTIONS OF THE VERTICAL IN MOUNTAINOUS TERRAIN*

H. Baussus von Luetzow
U.S. Army Topographic Laboratories
Fort Belvoir, Virginia 22060

ABSTRACT: The paper first addresses the interpolation of gravity anomalies in mountainous terrain, to be represented as the sum of a "signal" variable with a quasi-stationary estimation structure and a computable "noise" variable without a stationary character. It then develops the particular solution of the boundary value problem of physical geodesy which permits a similar representation of deflections of the vertical and draws some conclusions concerning the inapplicability of Molodensky's series approach and of the collocation method for the accurate determination of vertical deflections from unmodified gravity anomalies in mountainous terrain. Thereafter, it discusses the estimation of signal-type deflections of the vertical by means of spatial covariance functions, i.e., by a linear regression technique called statistical collocation in physical geodesy, and provides first order expansions of planar covariance functions.

1. INTRODUCTION. Deflection of the vertical components ξ and η play a role in the adjustment of geodetic networks, in the computation of height anomaly differences, and in the transformation of local coordinates into terrestrial coordinates. Short of a three-dimensional solution of the geodetic boundary value problem under consideration of mountainous terrain, deflection components and gravity anomalies Δg are also desirable for the numerical upward continuation of the first order derivatives of the anomalous gravity potential. The interpolation or estimation of gravity vector components in flat terrain is not inherently difficult. In mountainous terrain, gravity anomalies Δg are profitably modified to Faye anomalies Δg_F by means of terrain corrections C, to be followed by a transformation

*Paper, accepted for presentation at the VIth International Symposium on Geodetic Computations, Munich, W. Germany, Aug 31-Sep 5, 1981. To be published in symposium proceedings and in Proc. 25th Conference of Army Mathematicians, U.S. Army Research Office, Research Triangle Park, NC 27709.

to Bouguer anomalies Δg_B which permit an approximate two-dimensional interpolation. Isostatic gravity anomalies Δg_I, to be corrected by the indirect effect, would require a three-dimensional interpolation technique in the case of high accuracy. The problem of deflection estimation has been discussed by Heiskanen and Moritz [1967] and others, including the method by Molodensky et al. [1962] for the calculation of deflection differences in flat terrain, and the difficulty to interpolate ξ, η in rough mountainous terrain. Baussus von Luetzow [1980] addressed the optimal densification of deflections of the vertical in flat terrain with and without consideration of gravity anomalies and extended Molodensky's approach. Badekas and Mueller [1968] utilized Eötvös torsion balance measurement together with appropriate terrain corrections for the interpolation of vertical deflections, a time-consuming procedure and soon to be replaced by the employment of moving base gravity gradiometers. Regardless of these efforts, an effective ξ, η-estimation method applicable in mountainous terrain will still be valuable and may also aid deflection estimation under consideration of a series of discrete inertial measurements. Section 2 of this study addresses the interpolation of gravity anomalies in mountainous terrain. In section 3, the appropriate solution of the boundary value problem for vertical deflections is presented and reformulated for optimal deflection estimation of "signal" components of ξ and η and computation of topographic "noise" terms. The estimation of signal-type components by means of spatial collocation and the development of first order approximations of spatial covariance functions is the subject of section 4.

2. INTERPOLATION OF GRAVITY ANOMALIES. It is well known that an accurate analytical representation of free-air anomalies in pronounced mountainous terrain can only be achieved by a polynomial of high degree by means of Δg-data available in a network of high resolution. As a consequence, satisfactory linear interpolation requires small mesh sizes Δx, Δy. The following modified anomalies have been

useful for geodetic applications and the purpose of interpolation:

$$\Delta g_F = \Delta g + C \tag{1}$$

where C is the terrain correction, is called Faye anomaly.

$$\Delta g_B = g_F - bh + \frac{3\delta T}{2R} \tag{2}$$

is the modified Bouguer or complete topographic anomaly where $b = 0.112$ mgalm^{-1} is the Bouguer gradient, h is the elevation of terrain, δT is the potential of topographic masses, and $R = 6371$ Km is the earth's mean radius.

$$\Delta g_i = \Delta g_B + C_i + a\delta\zeta = \Delta g + C - bh + C_i + a\delta\zeta + r \tag{3}$$

is the isostatic anomaly valid for the compensated geoid with $a = 0.3086$ mgal m^{-1}, $\delta\zeta$ as the vertical separation between geoid and cogeoid, and r as a random error.

Equation (3) may be further written as

$$\Delta g = \Delta g_i + C_t + r \tag{4}$$

where C_t represents the aggregate of terms computable from the known topography. In a more general form, also applicable to the optimal estimation of vertical deflections, equation (4) is reformulated as

$$m = s + n + r \tag{5}$$

In this equation, m is a "message" variable, s is a "signal" variable, n is deterministic or computable "noise," and r is random-type noise.

Under consideration of a linear signal estimation structure, a signal can then be optimally estimated as

$$\hat{s}_e = L(m_i - n_i - r_i) \tag{6}$$

where L denotes a linear operator and the subscripts e and i refer to the estimation point P_e and measurement points P_i, respectively. The optimal measurement at P_e results as

$$\hat{m}_e = \hat{s}_e + n_e + r_e = L(m_i - n_i) + r_e - L(r_i) + n_e \tag{7}$$

The estimation error is

$$e(\hat{m}_e) = e(\hat{s}_e) + e[r_e - L(r_i)] \tag{8}$$

The corresponding estimation error resulting from the utilization of topographically unmodified measurements m_i is

$$\begin{aligned} e(m_e) &= \hat{s}_e - L(s_i) + r_e - L(r_i) + n_e - L(n_i) \\ &= e(\hat{s}_e) + e[r_e - L(r_i)] + n_e - L(n_i) \end{aligned} \tag{9}$$

Comparison of equation (9) with equation (8) shows that the non-optimal interpolation process is associated with a "topographic" estimation error $n_e - L(n_i)$ which becomes in general intolerable in moderate to rough mountainous terrain and thus induces the requirement of a fine mesh data grid.

The interpolation of isostatic anomalies by means of spatial collocation will be treated in conjunction with the interpolation of isostatic deflections of the vertical in section 4.

3. FORMULATION OF A VERTICAL DEFLECTION SOLUTION SUITABLE FOR OPTIMAL INTERPOLATION. Gravimetric-topographic solutions for the anomalous gravity potential and deflections of the vertical which inherently permit a "signal-noise" separation according to equation (5) have been established by Pellinen [1969], Moritz [1969], and Baussus von Luetzow [1971]. The latter emphasized that these essentially identical solutions are almost equivalent to those of Molodensky et al. [1962] and Brovar [1964], but are less data dependent, more direct from the computational view, and more advantageous for the utilization of artificial satellite data. The notations to

be used are the following:

$\zeta = T.\gamma^{-1}$	height anomaly
T	anomalous gravity potential
γ	normal gravity
$\xi = -(\frac{\partial\zeta}{\partial x})_{h=const.}$	prime vertical deflection
$\eta = -(\frac{\partial\zeta}{\partial y})_{h=const.}$	meridian vertical deflection
$h = h_P$	elevation of terrain referring to moving point P
h_A	elevation of terrain referring to fixed computation point A
$\beta_1 = \text{arc}\,\frac{\partial h}{\partial x}$	northern terrain inclination
$\beta_2 = \text{arc}\,\frac{\partial h}{\partial y}$	eastern terrain inclination
$\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$	derivatives taken along the local horizon in a northern and eastern direction
G	global mean gravity
α	azimuth angle counted clockwise from north
ψ	angle between the radius vectors $\vec{r}_A$ and $\vec{r}_P$ originating at the earth's spherical center
$S(\psi)$	Stokes' function
$k = 6.67.10^{-8}\,cm^3 g^{-1} sec^{-2}$	gravitational constant
$\rho = 2.67\ g\ cm^{-3}$	standard density
$R = 6371$ Km	earth's mean radius
$l_0 = 2R \sin\frac{\psi}{2}$	see Figure 1
$l = (r_A^2 + r_P^2 - 2r_A r_P \cos\psi)^{\frac{1}{2}}$	see Figure 1
σ	unit sphere (full solid angle)
g	measured gravity
$\Delta g = g-\gamma$	gravity anomaly
C	terrain correction
$\Delta g_F = \Delta g + C$	Faye anomaly

$b = 0.112$ mgal m^{-1}	Bouguer gradient
δT	potential of topographic masses
$\Delta g_B = \Delta g_F - bh + \frac{3\delta T}{2R}$	modified Bouguer or complete topographic anomaly

The geometry involving h_A, $h=h_P$, ψ, R, l_0, and l is evident from Figure 1 below.

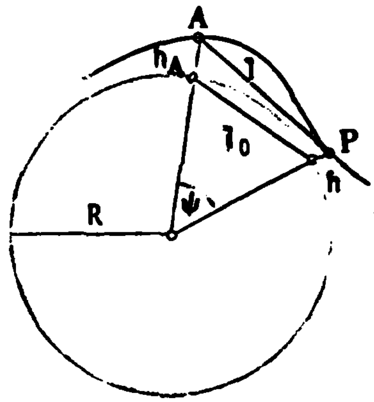

Figure 1

The established first order solution for the deflection components is

$$\begin{Bmatrix}\xi\\ \eta\end{Bmatrix} = \frac{R}{4\pi G}\iint_\sigma (\Delta g_F + \delta g_1 + G_1)\begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix}\frac{dS(\psi)}{d\psi}\,d\sigma + \begin{Bmatrix}\delta\xi\\ \delta\eta\end{Bmatrix} - \frac{\Delta g + G_1}{G}\begin{Bmatrix}\tan\beta_1\\ \tan\beta_2\end{Bmatrix} \tag{10}$$

where

$$\delta g_1 = \frac{3}{2}\,k\rho R\iint_\sigma \left(\ln\frac{l_1 + h - h_A}{l_0} - \frac{h - h_A}{l_0}\right)d\sigma,\quad l_1 = \sqrt{l_0^2 + (h - h_A)^2} \tag{11}$$

$$G_1 = \frac{R^2}{2\pi}\iint_\sigma \frac{\Delta g_B (h - h_A)}{l_0^3}\,d\sigma \tag{12}$$

$$\begin{Bmatrix}\delta\xi\\ \delta\eta\end{Bmatrix} = \frac{k\rho R^3}{G}\iint_\sigma \frac{h - h_A}{l_0^2}\left(\frac{1}{l_0} - \frac{1}{l_1}\right)\begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix}\sin\psi\,d\sigma \tag{13}$$

It should be noted that δg is in general very small and that the computation of $\delta g_1, \delta\xi$, and $\delta\eta$ requires only integrations over $\sigma_1 \ll 4\pi$. It has further to be emphasized that, according to Baarda [1979], the inclination angles β_1 and β_2 should not exceed 70°.

Equation (10) is now reformulated under consideration of

$$\Delta g_F = \Delta\hat{g} + (\Delta g_F - \Delta\hat{g}) = \Delta\hat{g} + \delta g_2 \tag{14}$$

$$\Delta g_B = \Delta\hat{g} + (\Delta g_B - \Delta\hat{g}) = \Delta\hat{g} + \delta\hat{g}_3 \tag{15}$$

In these equations, $\Delta\hat{g}$ is a signal variable, profitably the isostatic anomaly defined in equation (3). In comparison with δg_2, $\delta\hat{g}_3$ is a relatively smooth topographic quantity.

The substitutions (14) and (15) transform equation (10) into

$$\begin{Bmatrix}\xi\\ \eta\end{Bmatrix} = \frac{R}{4\pi G}\iint_\sigma [\Delta\hat{g} + G_1(\Delta\hat{g})] \begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix} \frac{dS(\psi)}{d\psi}\, d\sigma - \frac{\Delta\hat{g} + G_1(\Delta\hat{g})}{G} \begin{Bmatrix}\tan\beta_1\\ \tan\beta_2\end{Bmatrix}$$

$$+ \frac{R}{4\pi G}\iint_\sigma [\delta g_1 + \delta g_2 + G_1(\delta\hat{g}_3)] \begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix} \frac{dS(\psi)}{d\psi}\, d\sigma + \begin{Bmatrix}\delta\xi\\ \delta\eta\end{Bmatrix} + \frac{C - \delta g_2 - G_1(\Delta\hat{g}_3)}{G} \begin{Bmatrix}\tan\beta_1\\ \tan\beta_2\end{Bmatrix} \tag{16}$$

The first two terms of equation (16), involving the anomaly $\Delta\hat{g}$, represent the "signal" components of ξ and η. The following three terms constitute computable topographic "noise." Permitting for random-type errors r_ξ and r_η, equation (16) can be written in analogy with equations (4) and (5) as

$$\begin{Bmatrix}\xi\\ \eta\end{Bmatrix} = \begin{Bmatrix}\hat{\xi}\\ \hat{\eta}\end{Bmatrix} + \begin{Bmatrix}\delta\xi_t\\ \delta\eta_t\end{Bmatrix} + \begin{Bmatrix}r_\xi\\ r_\eta\end{Bmatrix} \tag{17}$$

The numerical determination of the three topographic terms of equation (16) is a complex task, which can, however, be accomplished without inherent difficulties by means of high-speed computers. In this respect, the integration area relating to the first topographic term can be considerably restricted. It appears that the last two topographic terms are particularly subject to rapid changes in mountainous terrain. Accurate interpolation is further favored if given and estimated deflections refer to points associated with small terrain inclinations.

In accordance with Moritz [1969], the second order correction for the height anomaly is

$$\delta\zeta^{(2)} = \frac{R}{4\pi}\iint_\sigma G_2(\Delta\hat{g} + \delta\hat{g}_3)\ S(\psi)d\sigma - \frac{R^2}{4\pi}\iint_\sigma (\Delta\hat{g} + \delta\hat{g}_3)\ \frac{(h-h_A)^2}{l_0^{\,3}}d\sigma \tag{18}$$

where

$$G_2 = \frac{R^2}{2\pi}\iint_\sigma \frac{h-h_A}{l_0^{\,3}}\ G_1(\Delta\hat{g} + \delta\hat{g}_3)d\sigma + (\Delta\hat{g} + \delta\hat{g}_3)\ tn^2\beta_m \tag{19}$$

Here, β_m represents the maximal terrain inclination.

The second order deflection corrections are then

$$\begin{Bmatrix}\delta\xi^{(2)}\\ \delta\eta^{(2)}\end{Bmatrix} = \begin{Bmatrix}\delta\xi^{(2)}(\Delta\hat{g})\\ \delta\eta^{(2)}(\Delta\hat{g})\end{Bmatrix} + \frac{R}{4\pi}\iint_\sigma G_2(\delta\hat{g}_3)\begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix}\frac{dS(\psi)}{d\psi}\,d\sigma$$

$$+ \frac{3R^3}{4\pi}\iint_\sigma \delta\hat{g}_3\ \frac{(h-h_A)^2}{l_0^{\,4}}\begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix}\frac{dl_0}{d\psi}\,d\sigma \tag{20}$$

Designating the integral terms of equation (20) as second order topographic corrections $\delta\xi_t^{(2)}$ and $\delta\eta_t^{(2)}$, equation (17) assumes the modified form

$$\begin{Bmatrix}\xi^{(2)}\\ \eta^{(2)}\end{Bmatrix} = \begin{Bmatrix}\hat{\xi}^{(2)}\\ \hat{\eta}^{(2)}\end{Bmatrix} + \begin{Bmatrix}\delta\xi_t\\ \delta\eta_t\end{Bmatrix} + \begin{Bmatrix}\delta\xi_t^{(2)}\\ \delta\eta_t^{(2)}\end{Bmatrix} + \begin{Bmatrix}r_\xi^{(2)}\\ r_\eta^{(2)}\end{Bmatrix} \tag{21}$$

Higher order topographic correction terms are not warranted because of a decreasing convergence radius in connection with higher derivatives, the assumption of a standard density or density uncertainties, respectively, and imperfect isostatic equilibrium. The structure of equation (21) clearly exhibits the fact that a highly accurate computation of $\xi^{(2)}$ and $\eta^{(2)}$ cannot be achieved by the exclusive utilization of free air anomalies Δg. For the same reason, iterative solutions of the integral equations for generalized surface densities by Molodensky et al. [1962] and Brovar [1964] and the series solution by Molodensky et al. [1962] in general do not converge in mountainous terrain. The latter permits for auxiliary boundary surfaces under utilization of a shrinking parameter $k \lessgtr 1_0$ and thus implies the possibility of analytical continuation with $\rho=0$. For the same reason, collocation solutions would only satisfactorily apply with respect to signal variables $\hat{\xi}$ and $\hat{\eta}$. The analytical upward continuation of the first derivatives of the anomalous gravity potential in mountainous terrain would require a supplemental approach.

4. <u>SIGNAL ESTIMATION BY STATISTICAL COLLOCATION AND FIRST ORDER EXPANSIONS OF PLANAR COVARIANCE FUNCTIONS</u>. As indicated by Baussus von Luetzow [1980], deflection differences in flat terrain may be advantageously determined by a combination of statistical collocation and Vening Meinesz formulae provided gravity anomalies are also available in sufficient density within a limited region. Four point deflection estimation errors with mesh sizes Δx = 5 km, 8 km, and 24 km were found to be, respectively, of the order 0.1 arcsec, 0.2 arcsec, and 1.0 arcsec in the case of estimators free of errors. Astrogeodetically determined deflections are, however, presently associated with errors of the order of 0.25 arcsec. In accordance herewith, it is advantageous to employ a relatively great number of estimators if this is feasible.

The signal variable to be estimated and representing either $\hat{\xi}$ or $\hat{\eta}$ may be

$\hat{x}_e$, and the estimators may be written $x_i + \delta_i$ with δ_i as a correlated measurement error independent of $\hat{x}_i$. Under the assumption of an existing signal and noise covariance structure the following linear regression equation can be formulated:

$$\hat{x}_e = \sum a_i(\hat{x}_i + \delta_i) = A_i(\hat{X}_i + \Delta_i) \tag{22}$$

It is then in matrix form, with bars indicating covariances,

$$\overline{\hat{x}_e \hat{X}_k} = A_i(\overline{\hat{X}_i \hat{X}_k} + \overline{\Delta_i \Delta_k}) = A_i N_{ik} \;, \qquad \begin{Bmatrix} i \\ k \end{Bmatrix} = 1, 2, \ldots, n \tag{23}$$

The solution for the regression coefficient matrix follows as

$$A_i = \overline{\hat{x}_e \; \hat{X}_K} \; N_{iK}^{-1} \tag{24}$$

In the case of given astrogeodetic vertical deflections, δ_i may be composed of astrogeodetic errors with a variance $(0.25 \text{ arcsec})^2$ and a correlated error partially caused by imperfect isostatic equilibrium.

With respect to the basis for the statistical collocation approach in physical geodesy, reference is made to Bjerhammar [1973], Grafarend [1973], Krarup [1969], Lauritzen [1971], Moritz [1970], and Tscherning [1973]. Of significance is that the spatial covariance function for the disturbing gravity potential has to satisfy Laplace's equation. Baussus von Luetzow [1973] emphasized the necessity to treat $\zeta - \bar{\zeta}$ as a correlated random variable where $\bar{\zeta}$ is a deterministic development of ζ in spherical harmonics of at least degree and order 15. In accordance herewith, the requirement of homogeneity prescribes and at least permits in practice a restriction to the planar approach in physical geodesy. Accordingly, $\frac{\partial T}{\partial z} = -\Delta g$, $\frac{\partial T}{\partial x}$, and $\frac{\partial T}{\partial y}$ are supposed to satisfy Laplace's equation. It is realized that the convenient requirements of homogeneity and quasi-flat terrain are only approximately satisfied.

Moritz [1976] and Nash and Jordan [1978] established specific T-covariance functions which can be expanded into space in a closed form. As has been shown by the latter authors, the spatial covariance function is

$$\phi_{TT}(r,\, z_1,\, z_2) = \int_0^{\infty} \rho F(\rho) e^{-\rho(z_1+z_2)} J_0(r\rho)d\rho \tag{25}$$

$$F(\rho) = \int_0^{\infty} r\phi_{TT}(r) J_0(r\rho)dr \tag{26}$$

In equations (25) and (26), J_0 is the zero-order Bessel function, r is the variable planar distance, z_1 and z_2 are the elevations of two points, and $\phi_{TT}(r)$ is the planar T - covariance function.

The spatial vertical deflection covariances may be derived from equation (25) in the form

$$\begin{Bmatrix} \phi_{\xi\xi}(r,\, z_1, z_2) \\ \phi_{\eta\eta}(r,\, z_1, z_2) \end{Bmatrix} = -(\gamma_1\gamma_2)^{-1} \begin{Bmatrix} \dfrac{\partial^2}{\partial x^2} \\ \dfrac{\partial^2}{\partial y^2} \end{Bmatrix} \phi_{TT}(r,\, z_1,\, z_2) \tag{27}$$

where $\gamma_1 = \gamma(z_1), \gamma_2 = \gamma(z_2)$.

For ϕ_{TT}- functions which permit the derivation of realistic vertical deflection covariance functions, the Hankel transforms (26) and (25) cannot be evaluated in closed form. As an example, Jordan's [1972] third-order Markov model

$$\phi_{TT}(r) = \text{var } T \left(1 + \frac{r}{D} + \frac{r^2}{3D^2}\right)e^{-\frac{r}{D}} \tag{28}$$

leads to the hypergeometric function when introduced in equation (26). Thereafter, $\phi_{TT}(r,\, z_1,\, z_2)$ only can be obtained by an extremely lengthy numerical integration. For this reason, it appears to be advantageous to develop first order approximations for spatial vertical deflection covariance functions under consideration of Jordan's [1972] planar results. In this respect it has to be emphasized that Jordan interchanged the conventional partial differentiations $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$.

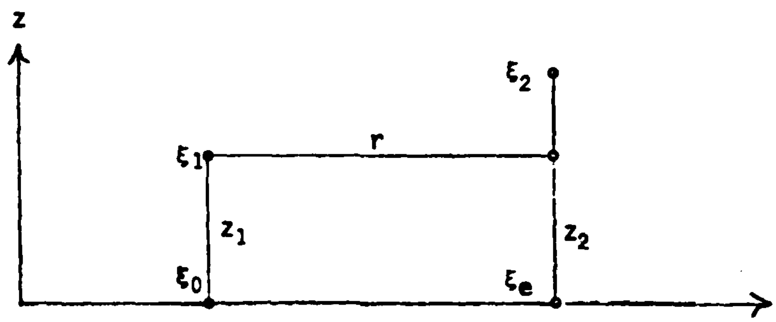

Figure 2

Under consideration of Figure 2, it is to the first order

$$\xi_1 = \xi_0 + \left(\frac{\partial \xi}{\partial z}\right)_0 z_1 \quad ; \quad \xi_2 = \xi_e + \left(\frac{\partial \xi}{\partial z}\right)_e z_2 \tag{29}$$

It is then, to the first order,

$$\overline{\xi_1 \xi_2} = \overline{\xi_0 \xi_e} + \overline{\xi_0 \frac{\partial \xi_e}{\partial z} z_2} + \overline{\xi_e \frac{\partial \xi_0}{\partial z} z_1} = \overline{\xi_0 \xi_e} + \overline{\xi_0 \frac{\partial \xi_e}{\partial z}}(z_1 + z_2) \tag{30}$$

It is further at level z = 0

$$\frac{\partial \xi}{\partial z} = -\frac{\partial}{\partial y} \frac{\partial \zeta}{\partial z} = -\frac{\partial}{\partial y}\left(-\frac{\Delta g}{G}\right) = \frac{1}{G} \frac{\partial \Delta g}{\partial y} \tag{31}$$

so that

$$\overline{\xi_0 \frac{\partial \xi_e}{\partial z}} = \overline{\xi_0 \cdot \frac{1}{G} \frac{\partial \Delta g_e}{\partial y}} = \frac{1}{G} \frac{\partial}{\partial y} \overline{\xi_0 \Delta g_e} \tag{32}$$

Under consideration of

$$\overline{\xi_0 \Delta g_e} = -\overline{\xi_e \Delta g_0} = -\frac{3\sigma_\xi \sigma_g}{\sqrt{2}} h(r) \frac{y}{r} \tag{33}$$

it is

$$\frac{\partial}{\partial y}\left[h(r)\frac{y}{r}\right] = \frac{\partial h(r)}{\partial y} \frac{y}{r} + h(r)\left(\frac{1}{r} - \frac{y^2}{r^3}\right) = \frac{\partial h}{\partial r} \cos^2\alpha + \frac{h}{r} \sin^2\alpha \tag{34}$$

The final results are hereafter

$$\overline{\xi_1\xi_2}\ (r,\ \alpha,\ z_1,\ z_2) = \phi_{\xi\xi} - \frac{3\sigma_\xi\sigma_g}{\sqrt{2}\ G}\left(\frac{h}{r}\sin^2\alpha + \frac{\partial h}{\partial r}\cos^2\alpha\right)(z_1 + z_2) \tag{35}$$

$$\overline{\eta_1\eta_2}\ (r,\ \alpha,\ z_1,\ z_2) = \phi_{\eta\eta} - \frac{3\sigma_\eta\sigma_g}{\sqrt{2}\ G}\left(\frac{h}{r}\cos^2\alpha + \frac{\partial h}{\partial r}\sin^2\alpha\right)(z_1 + z_2) \tag{36}$$

where $\phi_{\xi\xi}$ and $\phi_{\eta\eta}$ represent the planar covariance functions $\overline{\xi_o\xi_e}$ and $\overline{\eta_o\eta_e}$, respectively, and where $\sigma_g = \sigma_{\Delta g} = (\mathrm{var}\Delta g)^{\frac{1}{2}}$.

In analogy with equation (30), it is

$$\overline{\Delta g_1\ \Delta g_2} = \overline{\Delta g_0\ \Delta g_e} + \overline{\Delta g_0\ \frac{\partial \Delta g_e}{\partial z}}\ (z_1 + z_2) \tag{37}$$

Under utilization of the planar approximation

$$\frac{\partial \Delta g}{\partial z} = -G\left(\frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x}\right) \tag{38}$$

it is

$$-G\overline{\Delta g_0\left(\frac{\partial \xi_e}{\partial y} + \frac{\partial \eta_e}{\partial x}\right)} = -G\left(\frac{\partial}{\partial y}\ \overline{\Delta g_0\xi_e} + \frac{\partial}{\partial x}\ \overline{\Delta g_0\eta_e}\right) \tag{39}$$

With the aid of equation (33), equation (39) can be formulated as

$$-G\overline{\Delta g_0\left(\frac{\partial \xi_e}{\partial y} + \frac{\partial \eta_e}{\partial x}\right)} = -G\ \frac{3\sigma_\xi\sigma_g}{\sqrt{2}}\left(\frac{h}{r} + \frac{\partial h}{\partial r}\right) \tag{40}$$

Accordingly,

$$\overline{\Delta g_1\ \Delta g_2}\ (r,\ z_1,\ z_2) = \phi_{gg} - \frac{3G\sigma_\xi\sigma_g}{\sqrt{2}}\left(\frac{h}{r} + \frac{\partial h}{\partial r}\right)(z_1 + z_2) \tag{41}$$

where $\phi_{gg} = \overline{\Delta g_o \Delta g_e}$.

It is evident from equations (35), (36), and (41) that these represent convenient closed approximations of the three spatial covariance functions of particular interest. In general, the planar covariance functions should apply to the lowest z-level in a particular area of application.

5. CONCLUSION. The immediate accurate interpolation of gravity anomalies and deflections of the vertical in mountainous terrain is only possible from data provided in a grid of high resolution. Optimal interpolation from data given at points separated by distances of the order 5-10 km or from multiple data incorporating measurement noise with shorter spacing can be accomplished by an appropriate representation of gravity anomalies and deflections as a signal-noise process with nonstationary noise computable from the earth's topography. In the case of deflections, a special solution of the geodetic boundary value problem is required. As a first approximation, Faye anomalies may be used as signal variables. Isostatic anomalies modified by the indirect effect provide a greater degree of homogeneity and isotropy. Implementation of the theory requires the utilization of existing "isostatic" computer programs and the establishment of a supplemental program under consideration of furnished analytical solutions. Signal estimation has to be facilitated by the use of spatial covariance functions first order approximations of which may be computed relatively easy from planar covariance expressions. The optimal interpolation method in conjunction with the special solution for deflections indicates that iterative or series solutions of the boundary value problem of physical geodesy cannot be expected to converge in mountainous terrain. The method developed is of practical significance for the densification of gravity anomaly and astrogeodetic deflection networks in mountainous terrain and is also valuable or indispensable, respectively, for the optimal estimation of gravity anomalies and deflections from astrogeodetic and inertial data in mountainous areas.

REFERENCES

Baarda, W. 1979. A Connection Between Geometric and Gravimetric Geodesy, A First Sketch. Netherlands Geodetic Commission Public. on Geodesy, New Series, Vol. 6, Nr. 4.

Badekas, J., and I. Mueller. 1968. Interpolation of the Vertical Deflection from Horizontal Gravity Gradients. J. Geoph. Research, Vol. 73, No. 22, pp. 6869-6878.

Baussus von Luetzow, H. 1971. New Solution for the Anomalous Gravity Potential. J. Geoph. Research, Vol. 76, No. 20, pp. 4884-4891.

Baussus von Luetzow, H. 1973. A Review of Collocation Methods in Physical Geodesy and Meteorology. Paper, presented at the National Fall Meeting of the Am. Geoph. Union, San Francisco, CA, Dec 1973.

Baussus von Luetzow, H. 1980. Optimal Densification of Deflections of the Vertical by Means of Astrogeodetic and Gravity Anomaly Data. Paper, presented at the Spring Meeting of the Am. Geophysical Union, Toronto, Canada, May 1980.

Bjerhammar, A. 1973. A General Model for Optimal Prediction and Filtering in the Linear Space. Methoden und Verfahren der mathematischen Physik, Band 14, Bibliogr. Inst., Mannheim, W. Germany.

Brovar, V. V. 1964. On the Solutions of Molodensky's Boundary Value Problem, Bull. Geod., No. 72, Paris.

Grafarend, E. 1973. Geodetic Stochastic Processes. Methoden und Verfahren der mathematischen Physik, Band 14. Bibliogr. Inst., Mannheim, W. Germany.

Heiskanen, W., and H. Moritz. 1967. Physical Geodesy. W. H. Freeman and Co., San Francisco and London.

Jordan, S. K. 1972. Self-Consistent Statistical Models for the Gravity Anomaly, Vertical Deflections, and Undulations of the Geoid. J. Geoph. Research, Vol. 77, No. 20, pp. 3660-3670.

Krarup, T. 1969. A Contribution to the Mathematical Foundation of Geodesy. Public. No. 44. Geodetic Institute, Copenhagen, Denmark.

Lauritzen, S. 1971. The Probabilistic Background of Some Statistical Methods in Physical Geodesy. Public. No. 48. Geodetic Institute, Copenhagen, Denmark.

Molodensky, M. S., V. F. Eremeev, and M. I. Yurkina. 1962. Methods for study of the external gravitational field and figure of the earth. Transl. from Russian (1960). Israel Program for Scientific Translations, Jerusalem.

Moritz, H. 1969. Nonlinear Solutions of the Geodetic Boundary Value Problem. Rep. 126, Dept. of Geodetic Science, Ohio State University, Columbus, Ohio 43212.

Moritz, H. 1970. Least-Squares Estimation in Physical Geodesy. Rep. 130, Dept. of Geodetic Science, Ohio State University, Columbus, Ohio, 43212.

Moritz, H. 1976. Covariance Functions in Least-Squares Collocation. Rep. 240, Dept. of Geodetic Science, Ohio State University, Columbus, Ohio 43212.

Nash, R. A., and S. K. Jordan. 1978. Statistical Geodesy-An Engineering Perspective. Proc. IEEE, Vol. 66, No. 5, pp. 532-550.

Pellinen, L. P. 1969. On the Computations of Plumbline Deflections and Quasigeoidal Heights in Highlands (in Russion with English abstract), Tr. ZNIGAiK, 176, Nedra Press, Moscow.

Tscherning, C. C. 1973. Application of Collocation Determination of a Local Approximation to the Anomalous Potential of the Earth Using "Exact" Astro-Gravimetric Collocation. Methoden and Verfahren der mathematischen Physik, Band 14, Bibliogr. Inst., Mannheim, W. Germany.

DATE
ILME